

2011年中科大数学分析解答

题. 计算

$$(1) \lim_{x \rightarrow +\infty} x \left[\left(1 + \frac{1}{x} \right)^x - e \right]; (2) \int_0^{\frac{\pi}{2}} \sin^7 x dx.$$

解答. (1) 由Taylor展开式

$$\begin{aligned} \left(1 + \frac{1}{x} \right)^x &= e^{x \ln \left(1 + \frac{1}{x} \right)} = e^{1 - \frac{1}{2x} + o\left(\frac{1}{x}\right)} \\ &\rightarrow x \left[\left(1 + \frac{1}{x} \right)^x - e \right] = e x \left(e^{-\frac{1}{2x} + o\left(\frac{1}{x}\right)} - 1 \right) \\ &= e x \left[-\frac{1}{2x} - o\left(\frac{1}{x}\right) \right] \\ &\rightarrow -\frac{e}{2} (x \rightarrow +\infty). \end{aligned}$$