中科院 811量子力学 2013年考研真题答案解析

一、解：（1） $H = \frac{P^2}{2\mu} + V(r) = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r)$

定态薛定谔方程：$\hat{H}\psi = E\psi$

$$\therefore \left[-\frac{\hbar^2}{2\mu}\left(\frac{1}{r}\frac{\partial^2}{\partial r^2}r - \frac{l^2}{\hbar^2 r^2}\right) + V(r)\right]\psi = E\psi$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{2\mu}{\hbar^2}\left[(E - V(r)) - \frac{\hat{l}^2}{\hbar^2 r^2}\right]\psi = 0$$

令 $\psi = R_l(r)Y_{lm}(\theta,\varphi), l = 0,1,2,....,m = l, l-1,...,-l.$

$$\therefore \frac{1}{r}\frac{\partial^2}{\partial r^2}(rR_l Y_{lm}) + \frac{2\mu}{\hbar^2}\left[(E - V(r)) - \frac{\hat{l}^2}{\hbar^2 r^2}\right]R_l Y_{lm} = 0$$

$$Y_{lm}\frac{1}{r}\frac{d^2}{dr^2}(rR_l) + \frac{2\mu}{\hbar^2}\left[(E - V(r)) - \frac{l(l+1)}{r^2}\right]R_l Y_{lm} = 0$$

$$\therefore \frac{1}{r}\frac{d^2}{dr^2}(rR_l) + \frac{2\mu}{\hbar^2}\left[(E - V(r)) - \frac{l(l+1)}{r^2}\right]R_l = 0$$

即 $\frac{d^2}{dr^2}R_l(r) + \frac{2}{r}\frac{d}{dr}R_l(r) + \frac{2\mu}{\hbar^2}\left[(E - V(r)) - \frac{l(l+1)}{r^2}\right]R_l(r) = 0$

（2） 令 $R_l(r) = \frac{\chi_l(r)}{r}$，并将 $l = 0$ 代入上式中.

$$\frac{d^2}{dr^2}\frac{\chi_0(r)}{r} + \frac{2}{r}\frac{d}{dr}\frac{\chi_0(r)}{r} + \frac{2\mu}{\hbar^2}(E - V(r))\frac{\chi_0(r)}{r} = 0$$

$$\therefore \chi_0'' + \frac{2\mu}{\hbar^2}[E - V(r)]\chi_0 = 0$$

$$\because V = \begin{cases} 0, & r \le a \\ \infty, & r > a \end{cases}$$

$$\therefore \chi_0(0) = \chi_0(a) = 0$$

令 $\beta = \frac{\sqrt{2\mu E}}{\hbar}(E > 0)$，在 $0 < r < a$ 中有：