

2013年答案:

一. 写出运动方程:

$$U(t) = M \frac{d^2 x(t)}{dt^2} + m \left[ \frac{d^2 x(t)}{dt^2} + \frac{d^2 [L \sin(\varphi(t))]}{dt^2} \right]$$

$$mg \sin(\varphi(t)) = \left[ \frac{d^2 \varphi(t)}{dt^2} + m \frac{d^2 x(t)}{dt^2} \right] \cos(\varphi(t))$$

联立求解得

$$\begin{aligned} & [(M+m) - m^2 L \cos^2(\varphi(t))] \frac{d^2 \varphi(t)}{dt^2} \\ & + m^2 L \sin(\varphi(t)) \cos(\varphi(t)) \left[ \frac{d\varphi(t)}{dt} \right]^2 \\ & - (M+m) mg \sin(\varphi(t)) + m U(t) \cos(\varphi(t)) = 0. \end{aligned}$$

当  $\varphi(t)$  较小时, 取  $\sin(\varphi(t)) \approx \varphi(t)$

$$\cos(\varphi(t)) \approx 1$$

$$\text{得方程 } [(M+m) - m^2 L] \frac{d^2 \varphi(t)}{dt^2} - (M+m) mg \varphi(t)$$