

C_m 随温度的变化

$$C_{p,m} = a + bT + cT^2 + dT^3$$

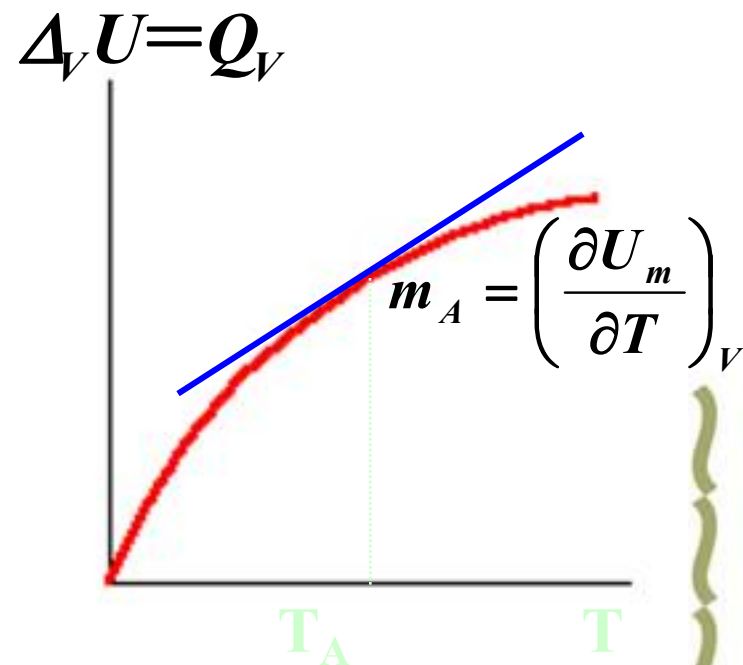
$$C_{p,m} = a' + b'T + c'T^{-2}$$

平均摩尔定压热容

$$\bar{C}_{p,m} = \frac{Q_{p,m}}{T_2 - T_1} = \frac{\int_{T_1}^{T_2} C_{p,m} dT}{T_2 - T_1}$$

$$\bar{C}_{p,m} = \frac{1}{2} [C_{p,m}(T_1) + C_{p,m}(T_2)]$$

$$\bar{C}_{p,m} = C_{p,m}(\bar{T}) \quad \bar{T} = \frac{1}{2}(T_1 + T_2)$$



$C_{V,m}$ 和 $C_{p,m}$ 的关系

$$\begin{aligned}C_{p,m} - C_{V,m} &= \left(\frac{\partial H_m}{\partial T} \right)_p - \left(\frac{\partial U_m}{\partial T} \right)_V = \left\{ \frac{\partial (U_m + pV_m)}{\partial T} \right\}_p - \left(\frac{\partial U_m}{\partial T} \right)_V \\&= \left(\frac{\partial U_m}{\partial T} \right)_p + p \left(\frac{\partial V_m}{\partial T} \right)_p - \left(\frac{\partial U_m}{\partial T} \right)_V\end{aligned}$$

由 $U_m = f(T, V)$ 得:

$$dU_m = \left(\frac{\partial U_m}{\partial T} \right)_V dT + \left(\frac{\partial U_m}{\partial V_m} \right)_T dV_m$$

恒压下, 可得:

$$\left(\frac{\partial U_m}{\partial T} \right)_p = \left(\frac{\partial U_m}{\partial T} \right)_V + \left(\frac{\partial U_m}{\partial V_m} \right)_T \left(\frac{\partial V_m}{\partial T} \right)_p$$

将上式代入 $(C_{p,m} - C_{V,m})$ 的式子中:

$$C_{p,m} - C_{V,m} = \left[\left(\frac{\partial U_m}{\partial V_m} \right)_T + p \right] \left(\frac{\partial V_m}{\partial T} \right)_p$$

恒压过程，温度升高体积膨胀，导致：

1. 系统内部分子势能升高，热力学能增加；
2. 对环境做功。

因此 $C_{p,m}$ 总是大于 $C_{V,m}$

对理想气体

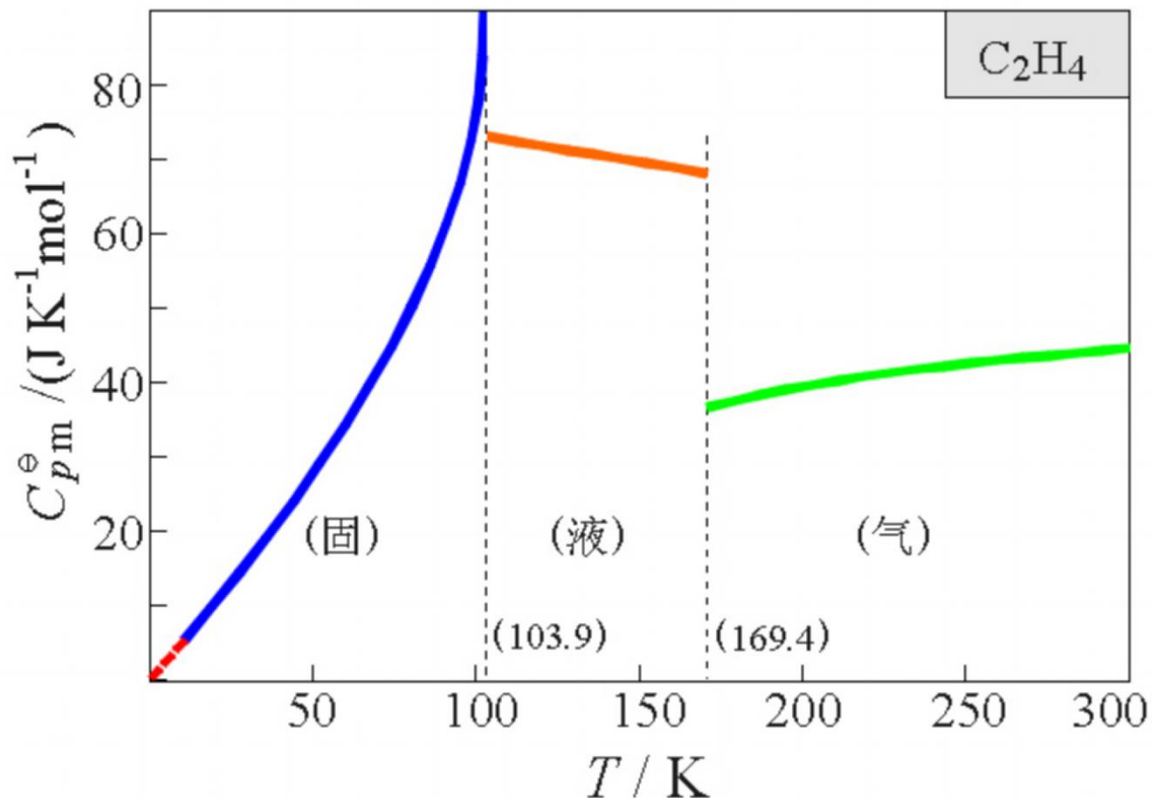
$$C_{p,m} - C_{V,m} = R$$

对液体与固体

$$C_{p,m} - C_{V,m} \approx 0$$

$$p \left(\frac{\partial V_m}{\partial T} \right)_p = p \left(\frac{\partial \left(\frac{RT}{p} \right)}{\partial T} \right)_p = p \frac{R}{p} = R$$
$$\left(\frac{\partial V_m}{\partial T} \right)_p \approx 0$$

- ◆热容是物质的特性
- ◆同一物质，聚集状态不同，热容不同
- ◆热容是温度的函数



◆理想气体的摩尔热容

单原子气体 $C_{V,m} = 3R/2$ $C_{p,m} = 5R/2$

双原子气体 $C_{V,m} = 5R/2$ $C_{p,m} = 7R/2$

2. 恒容变温及恒压变温过程热的计算

对于n一定的某系统进行单纯pVT变化

$$C_{V,m} \stackrel{\text{def}}{=} \frac{\delta Q_V}{dT} = \left(\frac{\partial U_m}{\partial T} \right)_V$$



$$Q_V = \Delta U_V = n \int_{T_1}^{T_2} C_{V,m} dT$$

$$C_{p,m} \stackrel{\text{def}}{=} \frac{\delta Q_p}{dT} = \left(\frac{\partial H_m}{\partial T} \right)_p$$



$$Q_p = \Delta H_p = n \int_{T_1}^{T_2} C_{p,m} dT$$

若 $C_{V,m}$ 为常数, 则 $Q_V = \Delta U_V = nC_{V,m}\Delta T$

若 $C_{p,m}$ 为常数, 则 $Q_p = \Delta H_p = nC_{p,m}\Delta T$

3. 凝聚系统变温过程

因为C

$$Q_V = Q_p = \Delta H_p = n \int_{T_1}^{T_2} C_{p,m} dT$$

§ 2.5 焦耳实验、理想气体的 U 与 H

1. 焦耳实验

2. 理想气体的热力学能与焓

$$W = 0 \quad Q = 0$$

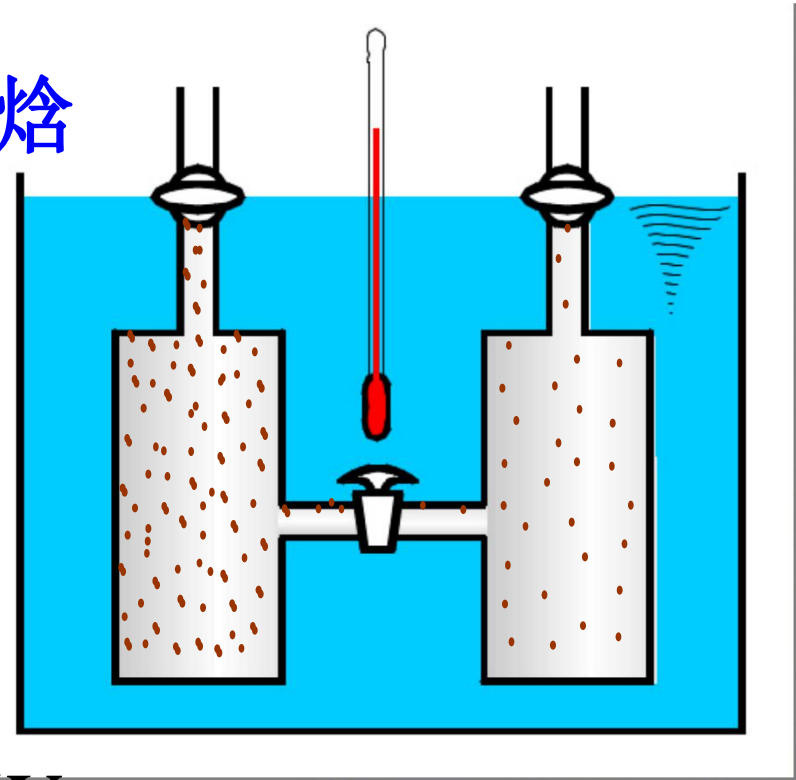
$$\Delta U = 0$$

$$dT = 0 \quad dU = 0$$

$$dV > 0 \quad dp < 0$$

$$dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV$$

$$\left(\frac{\partial U}{\partial V} \right)_T = 0 \quad \text{同理} \quad \left(\frac{\partial U}{\partial p} \right)_T = 0$$



理想气体 $U = f(T)$

对于无相变及化学变化的
理想气体等温过程: $\Delta U = 0$

$$H=U+pV$$

$$\left(\frac{\partial H}{\partial p}\right)_T = \left(\frac{\partial U}{\partial p}\right)_T + \left(\frac{\partial(pV)}{\partial p}\right)_T = 0 + \left(\frac{\partial(nRT)}{\partial p}\right)_T = 0$$

$$\text{同理}\left(\frac{\partial H}{\partial V}\right)_T = 0$$

对于无相变及化学变化的理想气体等温过程：

$$\Delta H = 0$$

对于理想气体 $U=f(T,V), H = f(T,P)$

$$dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV = \left(\frac{\partial U}{\partial T} \right)_V dT + 0$$

$$= n \left(\frac{\partial U_m}{\partial T} \right)_V dT = n C_{V,m} dT$$

$$\Delta U = n \int_{T_1}^{T_2} C_{V,m} dT$$

$$dH = \left(\frac{\partial H}{\partial T} \right)_p dT + \left(\frac{\partial H}{\partial p} \right)_T dp = \left(\frac{\partial H}{\partial T} \right)_p dT + 0$$

$$= n \left(\frac{\partial H_m}{\partial T} \right)_p dT = n C_{p,m} dT$$

$$\Delta H = n \int_{T_1}^{T_2} C_{p,m} dT$$

说明对于理想气体， ΔU 与 ΔH 的计算不再受过程恒容与恒压条件的限制。

❖ 例：将10.0g 523K、 $2.00 \times 10^5 \text{Pa}$ 的CO(g)等压降温到273K，计算此过程的Q、 ΔU 、 ΔH 。已知在此温度区间 $\bar{C}_{p,m}(\text{CO}, g) = 29.50 \text{J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$ 。

$$\text{解： } n = \frac{m}{M} = \frac{10.0}{28.01} \text{mol} = 0.375 \text{mol}$$

$$Q_p = \Delta H = n \bar{C}_{p,m} (T_2 - T_1) = -2.63 \text{kJ}$$

$$\begin{aligned} (1) \Delta U &= \Delta H - \Delta(pV) = \Delta H - \Delta(nRT) \\ &= \Delta H - nR\Delta T = -1.89 \text{kJ} \end{aligned}$$

$$(2) \bar{C}_{V,m} = \bar{C}_{p,m} - R = 21.19 \text{kJ} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$$

$$\Delta U = n \bar{C}_{V,m} (T_2 - T_1) = -1.89 \text{kJ}$$

等压过程

$$\Delta U_p = U_2 - U_1$$

初态

$$T_1 = 523\text{K}$$

$$P_1 = 2.00 \times 10^5 \text{Pa}$$

$$V_1 = 7.76 \times 10^{-3} \text{m}^3$$

U_1 、 H_1

末态 (1)

$$T_2 = 273\text{K}$$

$$P_1 = 2.00 \times 10^5 \text{Pa}$$

$$V_2 = 4.05 \times 10^{-3} \text{m}^3$$

U_2 、 H_2

$$\Delta U_T = 0$$

末态 (2)

$$T_2 = 273\text{K}$$

$$P_2 = 1.04 \times 10^5 \text{Pa}$$

$$V_1 = 7.76 \times 10^{-3} \text{m}^3$$

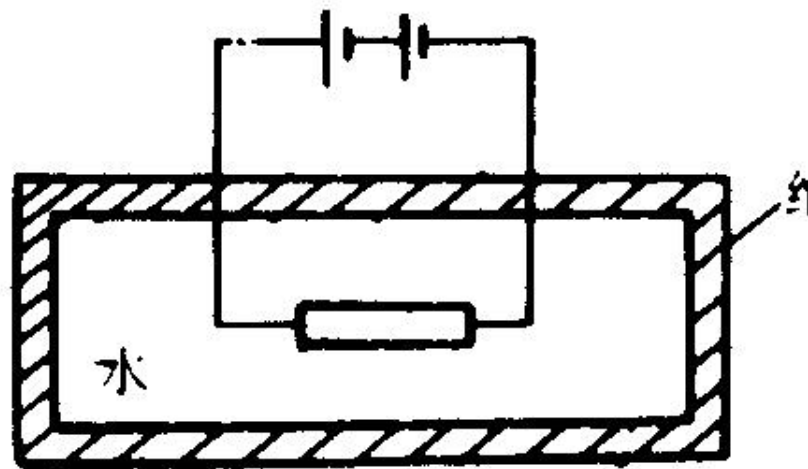
U_2' 、 H_2'

等容过程

$$\Delta U_V = U_2' - U_1$$

$$\Delta U_p = \Delta U_V = n\bar{C}_{V,m}(T_2 - T_1)$$

例1： 将一电炉丝浸入刚性绝热容器的水中，接上电源通电一段时间，试判断此过程中(a)水的热力学能，(b)水和电炉丝总的热力学能是增加、减少还是不变？



解： (a) $Q>0, W=0, \Delta U=Q+W>0$

(b) $Q=0, W>0, \Delta U=Q+W>0$

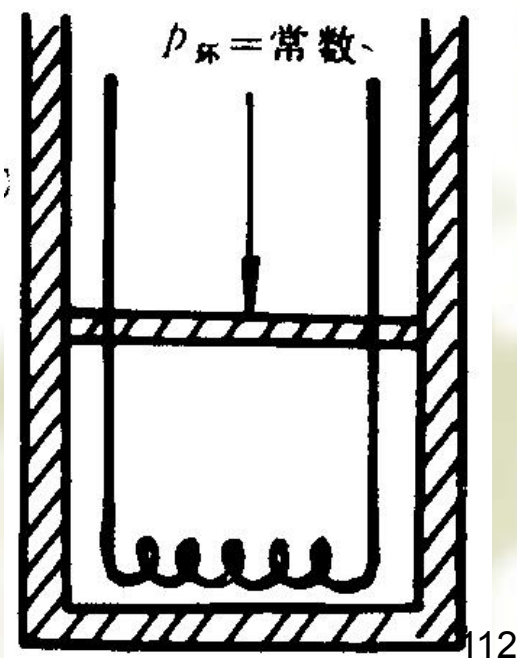
例2: 在一个带有无摩擦、无重量的绝热活塞的绝热气缸内充入一定量的气体。气缸内壁绕有电阻丝，活塞上方施以一恒定压力，并与缸内气体成平衡状态，如右图所示。现通入一微小电流，使气体缓慢膨胀。此过程为一等压过程，故 $Q_p = \Delta H$ ，而该系统为一绝热系统，则 $Q_p = 0$ ，所以此过程的 $\Delta H = 0$ 。此结论

(1) 以气体为系统:

$$\because W' = 0, Q > 0, \therefore Q_p = \Delta H > 0$$

(2) 以气体+电阻丝为系统:

$$\because W' > 0, Q = 0, \therefore Q_p \neq \Delta H$$



例3： 在炎热的夏天，有人提议打开室内正在运行的冰箱的门，以降低室温，你认为此建议可行吗？

$$Q = 0, W > 0 \Rightarrow \Delta U = Q + W > 0$$

$$\Delta U = C_{V,m}(T_2 - T_1) > 0 \Rightarrow T_2 > T_1$$

上次课主要内容

1. 恒容热、恒压热、焓

$$H=U+pV \quad Q_V=\Delta U \quad Q_p=\Delta H$$

2. 恒容变温及恒压变温过程热的计算

$$C_{V,m} \stackrel{\text{def}}{=} \frac{\delta Q_V}{dT} = \left(\frac{\partial U_m}{\partial T} \right)_V$$



$$Q_V = \Delta U_V = n \int_{T_1}^{T_2} C_{V,m} dT$$

$$C_{p,m} \stackrel{\text{def}}{=} \frac{\delta Q_p}{dT} = \left(\frac{\partial H_m}{\partial T} \right)_p$$



$$Q_p = \Delta H_p = n \int_{T_1}^{T_2} C_{p,m} dT$$

对理想气体 $C_{p,m} - C_{V,m} = R$

对液体与固体 $C_{p,m} - C_{V,m} \approx 0$

3. 理想气体的热力学能与焓

$$\left(\frac{\partial U}{\partial V}\right)_T = 0 \quad \left(\frac{\partial U}{\partial p}\right)_T = 0$$

$$\left(\frac{\partial H}{\partial V}\right)_T = 0 \quad \left(\frac{\partial H}{\partial p}\right)_T = 0$$

$$\Delta U = n \int_{T_1}^{T_2} C_{V,m} dT$$

$$\Delta H = n \int_{T_1}^{T_2} C_{p,m} dT$$

§ 2.6 可逆过程与绝热过程

1. 可逆体积功

$$W_a = -p_{\text{环}} \Delta V = -18.0 \text{ kJ}$$

$$W_b = -\Sigma p_{\text{环}} \Delta V = [-2 \times (12 - 6) - 1 \times (24 - 12)] \text{ kJ} = -24.0 \text{ kJ}$$

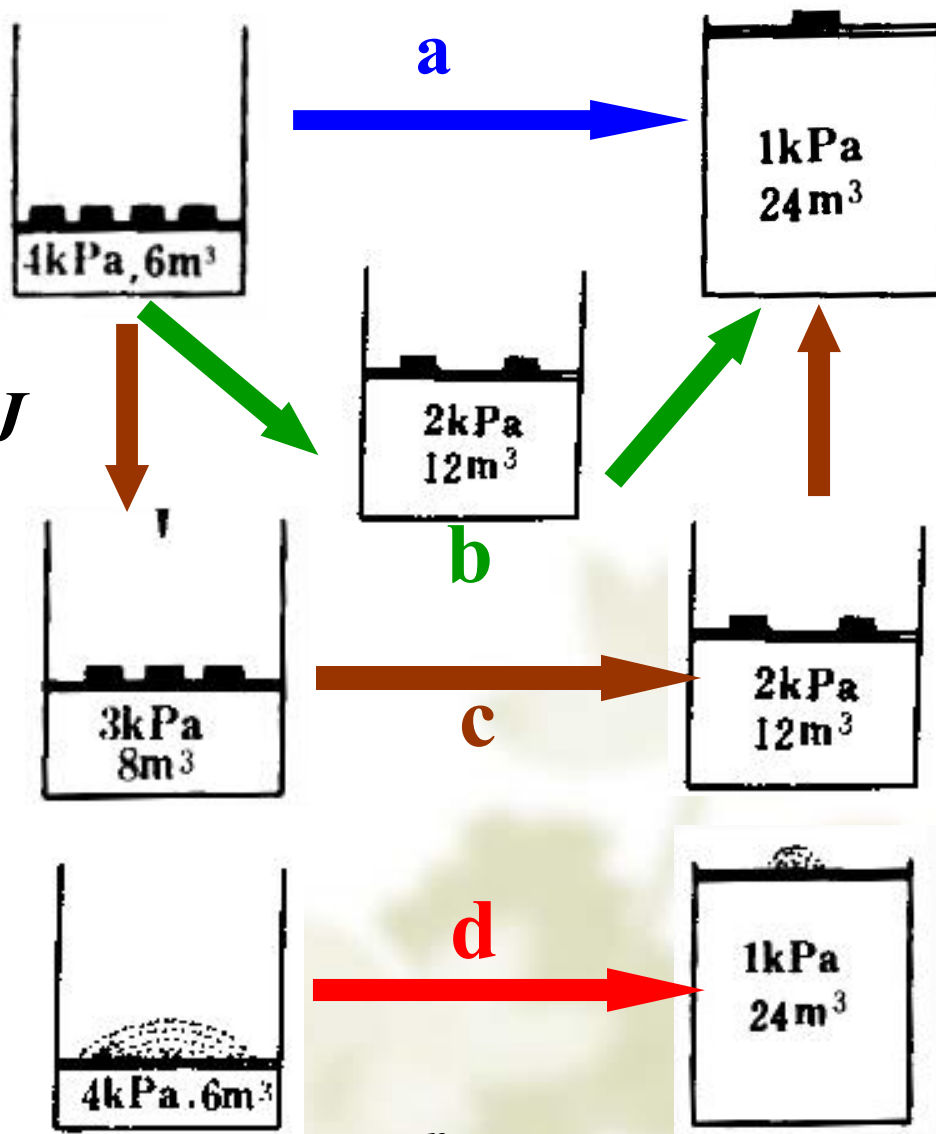
$$W_c = -26.0 \text{ kJ}$$

$$W_d = -\int_{V_1}^{V_2} p_{\text{环}} dV$$

$$= -\int_{V_1}^{V_2} (p - dp) dV$$

$$= -\int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{nRT}{V} dV$$

$$= nRT \ln \frac{V_1}{V_2} = nRT \ln \frac{p_2}{p_1}$$



$$W_d = p_1 V_1 \ln \frac{p_2}{p_1} = -33.3 \text{ kJ}$$

将以上过程逆向进行

$$W_{a'} = 72.0 \text{ kJ}$$

$$W_{b'} = 48.0 \text{ kJ}$$

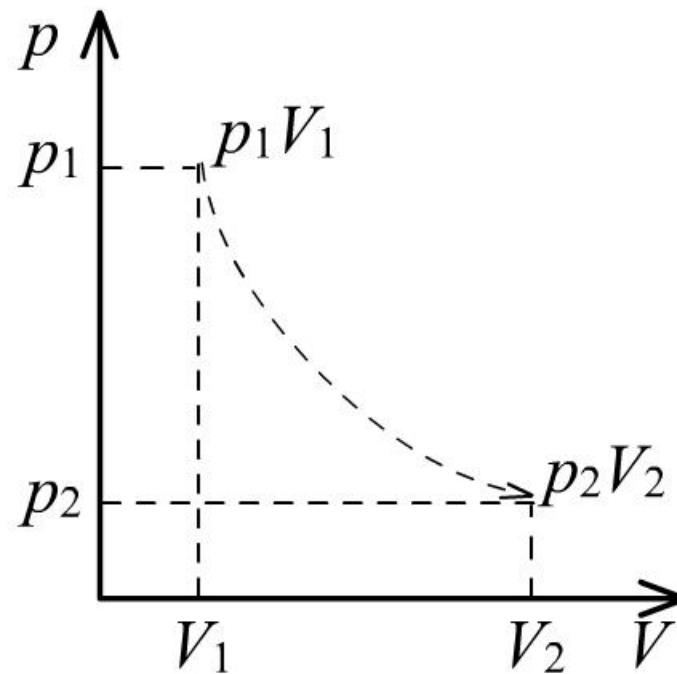
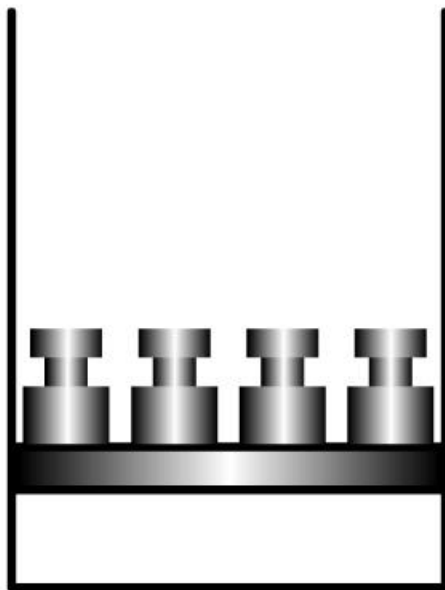
$$W_{c'} = 44.0 \text{ kJ}$$

$$W_{d'} = 33.3 \text{ kJ}$$

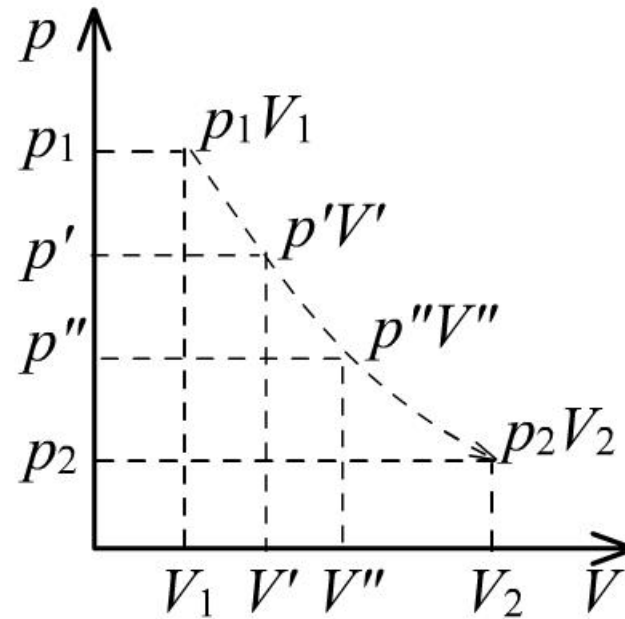
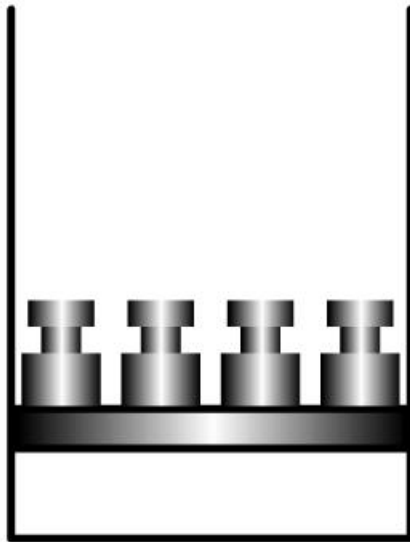
膨胀(压缩) 方式	一次	二次	三次	多次
W(膨胀)/kJ	-18	-24	-26	-33.3
W(压缩)/kJ	72	48	44	33.3
$\Sigma W/\text{kJ}$	54	24	18	0
$\Sigma Q/\text{kJ}$	-54	-24	-18	0

在同样的初末态之间进行的等温过程中，可逆膨胀时，系统对环境作最大功；可逆压缩时，环境对系统作最小功。

一次等外压膨胀



多次等外压膨胀



可逆膨胀

